

# The Role of Transformation Groups for the Ontology of Geometrical Objects

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## Abstract

Although we may ascribe to modern axiomatics the successful treatment of geometrical concepts with the least appeal to intuition, the link between mathematics and the physical world cannot and should not be denied, firstly because it reappears even in the purest process of thoughts, secondly, because of its importance in order to save the historical and psychological roots of the current geometrical methods. The problem posed here is to define the basic notions of elementary geometry (that of point and of straight line) saving both the need for rigour and the empirical genesis of mathematical objects. Felix Klein's *Erlanger Program* and Poincaré's booklet *On the foundations of geometry* give us the key notions for a plausible solution. For the first time, with Felix Klein's dissertation the concept of transformation group revealed its central role in organizing and defining different geometrical systems. It gives, for example, a precise meaning to the notion of *geometrical* property by means of the theory of invariants. Moreover, Poincaré's thought-provoking observations punctualized that the concept of group could play a major role in the genesis of geometry. According to Poincaré, the mind (*esprit*) is endowed with the a-priori structure of groups which is imposed upon the unstructured manifold of sense impressions, in order to give them the form of Euclidean space.

Thanks to the group of displacements we are able to abstract, from things materially different, the same mathematical object. We will show that it is possible, starting from Poincaré's considerations, to build an immanent realization of a given group. This achievement allows us to give a proper definition to our basic categories of mathematical objects, and even to retain those links between human experience and abstract theory, whose essential role for the comprehension of the development of geometry, far from being removed, must be emphasized.