Constraint Programming 2002: Assignment 3

Assignment 3-1

Answer the following questions.

1. What are the main characteristics of heuristic methods? What's the difference between a heuristic method and an exact method?
2. What is the main motivation for using heuristics?
3. Why are heuristic methods in general incomplete?

Assignment 3-2

Plauteaus and local minima are a major problem in the design of a heuristic search method.

1. What is a plateau? What is a local minimum? What's the difference?
2. Why is the existence of these regions in the search space a problem for heuristic methods?
3. Name three different plateau escape strategies and describe them, and discuss drawbacks and advantages of them.

Assignment 3-3

Assume a CSP $P = (Z, D, C)$ where

$$Z = \{x, y, z, w\}, D_{\boldsymbol{t}} = \{1, 2, 3, 4, 5\} \text{ where } \boldsymbol{t} \in Z \text{ and } C = \{\texttt{alldiff}(Z), x < y, y < z, z < w\}$$

Use the cost function $f(v) = \sum_{c \in C} f_c(v)$ where

$$f_{x<y}(v) = \begin{cases} 0 & \text{if } v(x) < v(y) \\ 1 & \text{otherwise} \end{cases}$$

and

$$f_{\text{alldiff}(Z)}(v) = |S| \text{ where } S = \{\{x, y\} \mid v(x) = v(y)\}$$

In other words, the cost is 1 for less-than constraints, and the number of sets of two variables taking the same value for `alldiff`-constraints. You'll minimize the cost since it is a measure of violation on the given assignment.

The current solution is $\{(x, 5), (y, 4), (z, 3), (w, 1)\}$. This can be specified as $[5, 4, 3, 1]$ using the order of the values given in the specification of Z. Give the neighbors and the cost of these for the current solution for the problem above using the three neighborhoods below.

1. 2-swap
2. 1-assign (restrict the neighborhood by showing what happens when changing x and z only)
3. 1-modify

Answer the following question with motivation.

1. Which neighborhood do you think is most suitable to the problem? Why is that? Which characteristic does this neighborhood have that makes it more suitable to this problem instance than the others?
2. Which one is least suitable?

Assignment 3-4

In this assignment we will attempt to solve a small instance of the well-known Travelling Salesman Problem (TSP) using local search.

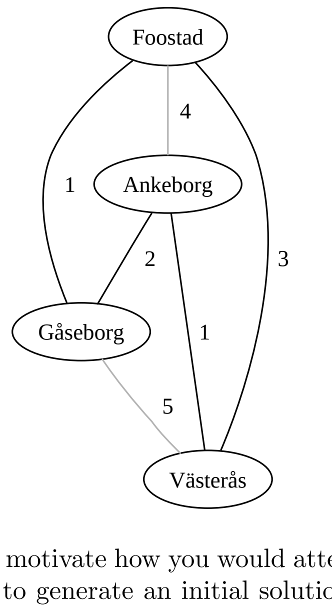
The problem can be stated informally as follows. Imagine a salesman who has to visit all the cities in a country. Because there is a cost (gasoline, etc.) associated with traveling between each pair of cities the salesman wants to minimize the distance travelled. The goal of the problem is then to minimize the length of the *tour*, which must visit all cities and begin and end in the same city.

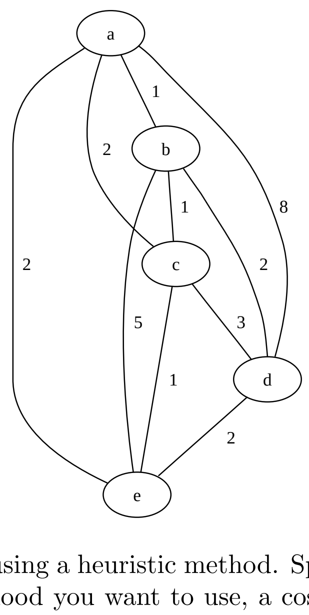
We now state TSP formally. First, we need to define some auxillary terms. A *path* in a graph (V, E) is a sequence $\langle v_0, v_1, \ldots, v_k \rangle$ of vertices $v_i \in V$ such that $(v_{i-1}, v_i) \in E$ for $i = 1, 2, \ldots, k$. The path *contains* the vertices $v_0, v_1, \ldots, v_k$ and the edges $(v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k)$. The path is *simple* if all vertices in the path are distinct. A path forms a *cycle* if $v_0 = v_k$ and the path contains at least one edge. A simple cycle that contains each vertex in V is a *hamiltonian cycle*.

We associate an integer cost $c(i, j)$ to each pair (i, j) of vertices. Formally, TSP is defined as a pair $\langle G, c \rangle$ where $G = (V, E)$ is a complete graph[1] and $c : V \times V \to \mathbb{Z}$ is a cost function from pairs of vertices to integers. The goal of TSP is to find a hamiltonian cycle p in G that visits all cities, and minimizes the sum of the costs of the edges in p. For the leftmost problem below, the path

$$\langle \text{Foostad}, \text{Gåseborg}, \text{Ankeborg}, \text{Västerås}, \text{Foostad} \rangle$$

represents a minimum-cost hamiltonian cycle with cost 7.





Discuss and motivate how you would attempt to solve TSP using a heuristic method. Specify the different states, how to generate an initial solution, which neighborhood you want to use, a cost function, a stop criteria, an improvement strategy and the metaheuristics (plateau escape mechanizms) you want to use and how you will use them in your algorithm. Show your approach by a single improvement step taken from the rightmost problem above.

[1] All pairs of vertices are connected with an edge